Extension of the Standard Model with the addition of the Weinberg operator*

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Even if the Standard Model of Particle Physics is still enjoying an unprecedented success, neutrinos remain the most enigmatic particles in the framework. The questions investigated nowadays address the problem of mass generation and of Dirac or Majorana type particles. The aim of this article is to present an effective method in which the Standard Model is extended to high mass dimensions via the addition of the lowest order non-renormalizable operator and to explore the phenomenology of the Neutrinoless Double β Decay to probe the properties of neutrinos at colliders.

INTRODUCTION

Neutrinos were discovered in 1956 [1], come in three flavours (electron, muon and tau), have very small but non-vanishing masses and only interact via the chiral weak force. The chirality condition leads to right-handed particles being unable to couple with the weak interaction gauge bosons but also to more fundamental questions about the nature of neutrinos. The same way spinors can be of Dirac or Majorana type, neutrinos can be of either nature, which has huge consequences on phenomenology. Neutrinos are the only particles whose masses cannot be generated by the usual Higgs mechanism [2], and despite the investigations of several see-saw mechanisms in which right-handed neutrinos acquire heavy masses, the possibility for neutrinos to be Majorana particles also allows us to imagine effective versions of the Standard Model in which new physics exists at very high energy. In this article, we explore the phenomenology of Majorana neutrinos through the addition of the Weinberg operator [3], the unique five mass dimensional operator of the effective Standard Model.

I. RENORMALIZABLE STANDARD MODEL

The renormalizable Standard Model [4] is a relativistic quantum field theory with interactions between particles described by a local Lagrangian. This Lagrangian is invariant under the local gauge symmetry defined by the group $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$; the gauge group is spontaneously broken into $\mathrm{SU}(3)_L\times\mathrm{U}(1)_Q$ as a result of the Higgs mechanism occuring due to the non-vanishing vacuum expectation value of a scalar field transforming as $\mathbf{(1,2)_{1/2}}$ under the local symmetry, and interactions are renormalizable, which means that only interactions up to the mass dimension 4 are allowed in the Lagrangian.

A. Field content

The model aims at describing all interactions at the microscopic level and classifying the existing elementary particles into two families: bosons and fermions. The three interactions it successfully describes are the electromagnetic, weak and strong interactions. The gauge group is:

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \tag{1}$$

where C, L, and Y refer respectively to color, left chirality and weak hypercharge. The $\mathrm{SU}(2)\times\mathrm{U}(1)$ factor in the gauge group comes from the pseudo-unified electroweak force, while the SU(3) factor corresponds to the strong force. For the electroweak force, we introduce ${\epsilon_{ij}}^k$ the structure constants of SU(2) and g_1 and g_2 the coupling constants associated to each group. The field strengths are:

$$\begin{aligned} W^i_{\ \mu\nu} &= \partial_\mu W^i_{\ \nu} - \partial_\nu W^i_{\ \mu} + g_2 {\epsilon_{jk}}^i W^j_{\ \mu} W^k_{\ \nu} \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{2}$$

where $W^i_{\ \mu}$ are three massless weak bosons and B_μ is a massless boson for the U(1) sector. For the strong force, we need eight massless gluons $g^a_{\ \mu}, a = 1,\ldots,8$, the structure constant ${f_{ab}}^c$ and the coupling constant g_3. The field strength is thus:

$$G^a_{\ \mu\nu} = \partial_\mu g^a_{\ \nu} - \partial_\nu g^a_{\ \mu} + g_3 {f_{ab}}^c g^b_{\ \mu} g^c_{\ \nu} \tag{3}$$

The matter sector is made of massless quarks and leptons described by fermionic fields with left chiral ones transforming as doublets and right chiral ones as singlets of $\mathrm{SU}(2)_L$. Quarks can have three colors corresponding to the fundamental representation of $\mathfrak{su}(3)$. Only left-handed spinors interact with the weak force meaning that weak interactions are chiral and that left- and right-handed fermions are in different representations of $\mathrm{SU}(2)\times\mathrm{U}(1)$. The index $i = 1,2,3$ for now refers to gauge states, but they can be associated to mass states (measured) after a transformation. The field content is:

1. left-handed quarks: $Q_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \qquad \mathbf{(3,2)_{1/6}}$

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2. right-handed quarks: $\overline{u}_{iR}$ $\mathbf{(3,1)_{2/3}}$ and $\overline{d}_{iR}$ $\mathbf{(3,1)_{-1/3}}$
3. left-handed leptons: we often use ℓ to denote the flavours of leptons, we do here because we will use i for the lepton mass eigenstates in the following sections: $L_{\ell L} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L$ $\mathbf{(1,2)_{-1/2}}$
4. right-handed leptons: $\overline{\ell}_R$ $\mathbf{(1,1)_{-1}}$

The last ingredient is the complex scalar doublet known as the HIGGS field: $H = \begin{pmatrix} H_+ \\ H_0 \end{pmatrix}$ $\mathbf{(1,2)_{1/2}}$. The quantum numbers give information about the transformations under the gauge groups and are used in the covariant derivatives

B. Lagrangian

To describe the behavior of the fields, one has to write the total Lagrangian of the Standard Model:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{fermion} + \mathcal{L}_{\text{HIGGS}} + \mathcal{L}_{\text{YUKAWA}} \tag{4}$$

The first term $\mathcal{L}_{\text{YM}}$ is the YANG-MILLS term and is defined using the field strength corresponding to the gauge fields of the theory:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i{}_{\mu\nu}W^{j\mu\nu}\delta_{ij} - \frac{1}{4}G^a{}_{\mu\nu}G^{b\mu\nu}\delta_{ab} \tag{5}$$

It describes the behavior of the gauge fields corresponding to the fundamental forces, the contributions including kinetic terms for the gauge fields and their self-interactions. The second contribution is $\mathcal{L}_{fermion}$ and it describes the dynamics of the fermions (quarks and leptons) and their interactions with the gauge fields. This part includes the kinetic terms for all the fermions as well as their gauge covariant derivatives. The covariant derivatives for each field depends on its hypercharge and on the interactions it feels. We use the PAULI matrices $\frac{1}{2}\sigma^i$ as generators in the fundamental representation of $\mathfrak{su}(2)$ and the GELL-MANN matrices $\frac{1}{2}\lambda^a$ for $\mathfrak{su}(3)$. Then, the various fields admit the following derivatives:

$$\begin{aligned}
D_\mu Q &= \left[\partial_\mu - ig_3\frac{1}{2}\lambda_a g^a{}_\mu - ig_2\frac{1}{2}\sigma_i W^i{}_\mu - ig_1\frac{1}{6}B_\mu\right]Q \\
D_\mu\overline{u} &= \left[\partial_\mu - ig_3\frac{1}{2}\lambda_a g^a{}_\mu - ig_1\frac{2}{3}B_\mu\right]\overline{u} \\
D_\mu\overline{d} &= \left[\partial_\mu - ig_3\frac{1}{2}\lambda_a g^a{}_\mu + ig_1\frac{1}{3}B_\mu\right]\overline{d} \\
D_\mu L &= \left[\partial_\mu - ig_2\frac{1}{2}\sigma_i W^i{}_\mu + ig_1\frac{1}{2}B_\mu\right]L \\
D_\mu\overline{\ell} &= \left[\partial_\mu + ig_1 B_\mu\right]\overline{\ell}
\end{aligned} \tag{6}$$

We can add to the description one last type of field to make a parallel with the following sections: if we assume that the neutrino is a MAJORANA fermion, we can add an iso-singlet neutrino field $\overline{N}_R$ that corresponds to a right-handed sterile neutrino, that is neutral and invariant under both weak and strong interactions. Its covariant derivative is then simply:

$$D_\mu\overline{N} = \partial_\mu\overline{N} \tag{7}$$

The fermion Lagrangian is then the sum of all these contributions for each generation:

$$\begin{aligned}
\mathcal{L}_{fermion} = {} & i\overline{Q_i}\bar{\sigma}^\mu D_\mu Q^i + iu^i\sigma^\mu D_\mu\overline{u_i} + id^i\sigma^\mu D_\mu\overline{d_i} \\
& + i\overline{L_i}\bar{\sigma}^\mu D_\mu L^i + ie^i\sigma^\mu D_\mu\overline{e_i} + iN^i\sigma^\mu D_\mu\overline{N_i}
\end{aligned} \tag{8}$$

The following term $\mathcal{L}_{\text{HIGGS}}$ is put by hand because of physical requirements and corresponds to the scalar potential of the HIGGS field. In order for the theory to be renormalizable, the potential is chosen to be:

$$\mathcal{L}_{\text{HIGGS}} = \mu^2 H^\dagger H + \lambda\left(H^\dagger H\right)^2 \tag{9}$$

This is the potential that will lead to the HIGGS mechanism and then the generation of the masses of the gauge bosons once we go to the vacuum state with $\mu^2 < 0$. What we still have to take into account are the interactions between the HIGGS field and the fermions. The fermions will develop a mass as a consequence of symmetry breaking by coupling with both the vacuum expectation value and the dynamical field. To be gauge invariant, these interactions are taken into account into YUKAWA couplings [5] between a left-handed fermion, a right-handed fermion and a scalar field:

$$\begin{aligned}
\mathcal{L}_{\text{YUKAWA}} = {} & -(y^e)^i{}_j\overline{e_i}\cdot L^j H^\dagger - (y^u)^i{}_j\overline{u_i}\cdot Q^j H^\dagger \\
& - (y^d)^i{}_j\overline{d_i}\cdot Q^j H^\dagger - (y^N)^i{}_j\overline{N_i}\cdot L^j H^\dagger + h.c
\end{aligned} \tag{10}$$

The coefficients are the YUKAWA coefficients and contain the coupling constants governing the strength of interactions between the HIGGS field and fermions. These matrices are three-dimensional complex matrices for each type of fermion. Then, when symmetry breaking occurs, the HIGGS field is replaced by its expectation value and the dynamical field, and the mass of the fermions will be generated.

II. DIRAC AND MAJORANA MASSES

In order to properly describe neutrinos, the first question to answer is about the type of spinor to use. A spin half particle can be described using a spinor, that obeys the DIRAC equation and has four independent components corresponding to particles and antiparticles with two possible helicities [6]. A spinor is defined by the way it transforms under the LORENTZ group. Experiments tell us that only left-handed neutrinos and right-handed antineutrinos are involved in weak interactions, meaning that a two-component Weyl spinor should in principle be sufficient to describe them. The problem is that Majorana spinors also fit the job, and the two have very different consequences on phenomenology.

A. Reminders about spinors

A particle with an electric charge is easily distinguishable from its antiparticle, but neutrinos are neutral so the discussion is non-trivial. A particle that is different from its antiparticle is described by a Dirac spinor, with independent degrees of freedom for the two pairs of components, while a particle that is its own antiparticle will be referred to as a Majorana particle. As a result, all its quantum numbers vanish and this requires the lepton and baryon number conservation to be violated. We define properly here Weyl, Dirac and Majorana spinors.

Consider the Dirac matrices in the usual representation $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$, closing a Clifford algebra. As a direct consequence, the matrices $\gamma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$ verify a Lorentz algebra that directly gives us information on the Lorentz transformations. A Dirac spinor is an object denoted ψ_D that transforms under the Lorentz group using the $\gamma^{\mu\nu}$ matrices which generate a reducible spinor representation of the Lorentz group [7].

$$\gamma^{\mu\nu} = \frac{1}{4}\begin{pmatrix} \sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu & 0 \\ 0 & \bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu \end{pmatrix} = \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{pmatrix} \tag{11}$$

The $\sigma^{\mu\nu}$ matrices will be used to transform Weyl spinors. To define them, we introduce the fifth Dirac matrix $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ which satisfies:

$$\gamma^{5\dagger} = \gamma^5, \qquad (\gamma^5)^2 = 1, \qquad \left[\gamma^5, \gamma^{\mu\nu}\right] = 0 \tag{12}$$

meaning we can build a complete set of orthogonal projector defining:

$$P_L = \frac{1}{2}(1-\gamma^5), \qquad P_R = \frac{1}{2}(1+\gamma^5), \qquad P_L + P_R = 1 \tag{13}$$

that acts on a Dirac spinor as:

$$P_L\psi_D = \begin{pmatrix} \lambda_L \\ 0 \end{pmatrix}, \qquad P_R\psi_D = \begin{pmatrix} 0 \\ \overline{\chi}_R \end{pmatrix} \tag{14}$$

These are Weyl spinors and the two projectors are called chirality projectors. Any fermionic degrees of freedom can be described equally well using a left-handed Weyl spinor or a right-handed one. By convention, all names of fermion fields are chosen so that left-handed Weyl spinors do not carry daggers and right-handed Weyl spinors do. It is overwhelmingly convenient to employ two-component Weyl spinor notation for fermions, rather than four-component Dirac spinors: the Standard Model Lagrangian violates parity, meaning that each Dirac fermion has left- and right-handed parts with completely different electroweak gauge interactions. If one used four-component spinor notation, then there would be left- and right-handed projection operators all over the computations. The two-component Weyl fermion notation has the advantage of treating fermionic degrees of freedom with different gauge quantum numbers separately from the start.

Majorana spinors (denoted ψ_M) are a peculiar kind of four-component spinors that can be obtained from the Dirac one by imposing a constraint:

$$\psi_M = \begin{pmatrix} \psi_L \\ \overline{\psi}_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ i\sigma^2\psi_L^* \end{pmatrix} \tag{15}$$

It has the same number of degrees of freedom as a Weyl spinor although it is written in the form of a Dirac spinor. Given a Dirac spinor, charge conjugation allows to define a new Dirac spinor:

$${\psi_D}^c = \begin{pmatrix} -i\sigma^2\psi_R^* \\ i\sigma^2\psi_L^* \end{pmatrix} = -i\begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}\psi_D^* \tag{16}$$

and a Majorana spinor is invariant under such an operation: $\psi_M = \psi_M^c$. We thus see that Majorana fermions are very similar to Weyl fermions, but they must satisfy a reality condition and they must be invariant under charge conjugation. The charge conjugation operation can seem confusing for a neutrino, but the neutral charge of these particles makes them the only candidates for Majorana fermions in the Standard Model as the charge does not allow to distinguish them. Let's consider a particle described by a field ψ, function of all spacetime coordinates. The operation of charge conjugation transforms a left-handed particle into a left-handed antiparticle, and the same for the right-handed, meaning that it leaves the helicity and chirality unaffected:

$$\left(P_{L,R}\psi\right)^c = P_{L,R}\psi^c = {\psi^c}_{L,R} = \left(\psi_{R,L}\right)^c \tag{17}$$

A confusion can come from the notation: $\psi_{L,R}$ and ${\psi^c}_{L,R}$ have the same helicities but we deal with a particle and an antiparticle, so this is not directly the charge conjugate but the charge-parity conjugate. We then refer to $\psi_{L,R}$ and ${\psi^c}_{L,R}$ as CP-conjugate. Parity is defined as usual as flipping the sign of all space coordinates, with fermions and antifermions having opposite parities.

B. Mass definition

In the Standard Model, neutrinos are assumed to be exactly massless, but this constraint was shattered by the Super-Kamiokande experiment in 1998. The experiment was able to detect electron and muon neutrinos, but what they observed was that neutrinos seemed to disappear in the detector: the number of expected neutrinos to be detected was far lower than the theoretical prevision, and the hypothesis that was formulated was that neutrinos could change flavour while propagating. The probability of a neutrino changing type is related to the distance travelled by the neutrino and its energy. Neutrino oscillation [8] arises from mixing between the flavour and mass eigenstates: the flavour eigenstates correspond to ν_e, ν_μ and ν_τ while the mass eigenstates correspond to ν_1, ν_2, and ν_3. Flavour eigenstates are linear combinations of mass eigenstates, and since the different mass eigenstates have different masses, they can propagate at different speeds. This implies that the phase difference between the mass states changes, resulting in a different linear combination and thus a different flavour state. Neutrinos are thus massive particles, meaning we have to define mass terms in any Lagrangian describing them, and choose the proper mass depending on the type of spinor. Consider only one flavour, the DIRAC Lagrangian is easily obtained from the DIRAC equation [6]:

$$\mathcal{L}_D = \overline{\psi}\left(i\gamma^\mu\partial_\mu - m_D\right)\psi, \qquad \overline{\psi} = \psi^\dagger\gamma^0 \tag{18}$$

A general spinor ψ can be decomposed into its left- and right-handed WEYL components $\psi = \psi_L + \psi_R$ with $\psi_L = \lambda_L$ and $\psi_R = \overline{\chi}_R$. The mass term $m_D\overline{\psi}\psi$ is called a DIRAC mass term. The DIRAC action we define by integration over spacetime is invariant under a global U(1) transformation of ψ_L and ψ_R simultaneously:

$$\psi_L \longrightarrow e^{i\alpha}\psi_L, \qquad \psi_R \longrightarrow e^{i\alpha}\psi_R \tag{19}$$

where α is the general parameter of the transformation. For MAJORANA spinors, ψ_L and ψ_R are not independent so if ψ_L transforms, then automatically $\psi_R = i\sigma^2\psi_L^*$ transforms as $e^{-i\alpha}\psi_R$. It is then impossible to define on a MAJORANA spinor a U(1) transformation under which the two components transform the same way at the same time. In other words, the MAJORANA equation:

$$i\bar{\sigma}^\mu\partial_\mu\psi_L - im_M\sigma^2\psi_L^* = 0 \tag{20}$$

is not invariant under global U(1) symmetries, and this means that a spin half particle carrying a U(1) conserved charge cannot have a MAJORANA mass. Because we do not know if neutrinos are DIRAC or MAJORANA fermions, the same goes for the mass: a DIRAC mass would imply that in addition with the left-handed neutrino, there exists a right-handed neutrino. However, these right-handed neutrinos are not seen in weak interactions and so if they exist they must be sterile or very heavy. The other possibility is then that neutrinos are described by purely left-handed fields and have MAJORANA masses.

Consider a free field without interaction that is described by the DIRAC equation. The DIRAC mass term is therefore $m_D\overline{\psi}\psi$, with the combination of spinors being LORENTZ invariant and hermitian. The Lagrangian has to be real, meaning that the DIRAC mass is real and the mass term couples left- and right-handed components:

$$m_D\overline{\psi}\psi = m_D\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right) \tag{21}$$

If neutrinos are DIRAC fermions, this mass is produced by the HIGGS mechanism. If they are MAJORANA fermions, the mass term is defined the following way [9]:

$$\begin{aligned}\mathcal{L}_M &= \frac{1}{2}\left(m_M\overline{\psi}\psi^c + m_M^*\overline{\psi^c}\psi\right) = \frac{1}{2}m_M\overline{\psi}\psi^c + h.c \\ &= \left[\frac{1}{2}m_L\overline{\psi}_L{\psi^c}_R + h.c\right] + \left[\frac{1}{2}m_R\overline{\psi^c}_L\psi_R + h.c\right]\end{aligned} \tag{22}$$

The problem is that nothing prevents us to deal with particles that are a statistical mixture of both DIRAC and MAJORANA type, so the most general Lagrangian for a DIRAC-MAJORANA mass term is:

$$2\mathcal{L} = \begin{pmatrix}\overline{\psi}_L & \overline{\psi^c}_L\end{pmatrix}\begin{pmatrix}m_L & m_D \\ m_D & m_R\end{pmatrix}\begin{pmatrix}{\psi^c}_R \\ \psi_R\end{pmatrix} + h.c \tag{23}$$

with the mass matrix admitting two eigenvalues:

$$m_{1,2} = \frac{1}{2}\left(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right) \tag{24}$$

These masses are taken to be positive. By defining the mixing angle $\tan(2\theta) = \frac{2m_D}{m_R - m_L}$ representing the rotation in state space to go from interaction to mass eigenstates, we can distinguish several interesting cases:

1. $\theta = 45°$: $m_L = m_R = 0$ so the eigenvalues of the mass matrix become $m_{1,2} = m_D$. This is a degenerate case corresponding to a pure DIRAC mass term. If the angle is slightly off and we have $\theta \sim 45°$: $m_D \gg m_L, m_R$ meaning that the eigenmasses are $m_{1,2} \sim m_D$ and that the fields $\phi_{1,2}$ are almost degenerate. This is a pseudo-DIRAC case.

2. $\theta = 0°$: $m_D = 0$ so the eigenmasses are purely of MAJORANA type $m_{1,2} = m_{L,R}$ and we have a pure MAJORANA case with associated neutrinos. Then again, if $\theta \sim 0°$: $m_R \gg m_D, m_L$ and this is a very interesting scenario because it involves a very light left-handed neutrino as well as a very heavy right-handed neutrino yet undetected. This is the basis for the different see-saw models where the masses of the left-handed and right-handed neutrinos are inversely proportional.

There exist three main types of scenario in which left-handed neutrinos are very light while right-handed are very massive [10]. To explain why we only observe left-handed neutrinos, we could simply lack energy to be able to probe the mass of the right-handed component, and these see-saw mechanisms are contained into the WEINBERG operator.

III. EFFECTIVE THEORY OF THE STANDARD MODEL

An effective field theory is a type of approximation for an underlying theory that takes into account the appropriate degrees of freedom to describe the physical phenomena occurring at a chosen energy scale while ignoring substructures and degrees of freedom at higher energies [11]. In our case, we will approximate a physical system by integrating out the degrees of freedom that are not relevant in a given experimental setting, and instead, these are traded for a set of effective interactions between the remaining degrees of freedom.

A. Renormalization condition

The aim of any theory is to make predictions on decay rates and cross-sections, and this is achieved using FEYNMAN diagrams computations relying on a set of integrals emerging from a diagram. These diagrams correspond to different orders in the interaction, and renormalizability is a crucial criterion because it ensures that all infinities arising from loop diagrams can be absorbed into a finite number of redefinitions of the parameters (couplings, masses, and fields). For a theory to be renormalizable, we say that the terms in the Lagrangian must have mass dimension smaller or equal to four in four-dimensional spacetime [12]. The Standard Model comes with additional conservation laws that a physical interaction process has to verify like charge conservation, energy conservation, isospin conservation, but two of them are not fundamental, and these are the lepton and baryon number conservation. The problem with these principles is that they cannot come from unbroken local symmetries, and thus are understood as accidental symmetries of the gauge group that can be broken at higher energies.

For a reaction to conserve lepton number, the sum of the lepton numbers before and after the reaction must be equal. This conservation applies separately to each type of lepton and this point is broken by neutrino oscillation. Neutrino oscillation states that a neutrino created with a specific lepton flavour can later be measured to have a different flavour, meaning that at least locally, the lepton number conservation does not hold as the neutrino does not have a single flavour. For a reaction to conserve baryon number, the sum of the baryon numbers before and after the reaction must be equal. This law implies that baryons cannot be created or destroyed without simultaneously creating or destroying an equivalent number of antibaryons. The apparent excess of baryons over antibaryons in the Universe provides a positive clue that some sort of physical processes may have actually violated baryon number conservation at very high energy and is now highly suppressed [3].

The Standard Model does not contain operators that can change the total baryon or lepton number. For instance, terms like $qqq\ell$ or $\ell\ell\ell\ell$ (where q is a quark and ℓ is a lepton) are not allowed because they would violate gauge invariance or renormalizability. Higher-dimensional operators could violate these symmetries, but because the dimension four Standard Model does not contain any of those terms, these numbers are conserved in all interactions without imposing it. This conservation is an accidental consequence of the allowed interactions and gauge symmetries of the Standard Model.

B. Weinberg operator

The Standard Model cannot be the final answer because it cannot predict all the masses or find good candidates for dark matter, the only thing we have to keep in mind is that no experimental data for now has been able to indicate that there are signs of new physics, at least up to energies of a few GeV. This seems to indicate that at these energies, the fundamental degrees of freedom are those of the Standard Model alone, and so it is reasonable to assume that new particles from beyond the Standard Model are much heavier than what we measure. If this assumption is correct, physics at the weak scale can be adequately described using effective field theory methods. By considering an effective theory of the Standard Model, the only difference is that interactions with arbitrary large mass dimensions are allowed. These interactions can be organized in a systematic expansion in the operator dimensions [13]:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_i^{n_5} C_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2}\sum_i^{n_6} C_i^{(6)} O_i^{(6)} + \dots \\ &= \mathcal{L}_{\text{SM}} + \sum_D \left[\frac{1}{\Lambda^{D-4}}\sum_i^{n_D} C_i^{(D)} O_i^{(D)}\right]\end{aligned} \tag{25}$$

Each $O_i^{(D)}$ is a gauge invariant operator of mass dimension D that is built using the Standard Model field content (a scalar has mass dimension one, a vector and a spinor have mass dimension 1/2), n_D is the number of such operators depending on the dimension and $C_i^{(D)}$ are called WILSON coefficients and taken to be free parameters for now. All the possible effects of high energy new physics are supposed to be encoded into these new $D > 4$ operators, and to be consistent with weak effects, these interactions are suppressed by appropriate powers of a mass scale Λ. What we assume about this mass scale is that it is greater than the vacuum expectation value of the HIGGS field.

For $D = 5$, there is a unique operator we can define, which is called the WEINBERG operator, which allows to generate MAJORANA masses after electroweak breaking:

$$\mathcal{L}_5 = \frac{C_{\ell\ell'}^{(5)}}{\Lambda}\left[H \cdot \overline{L_\ell^c}\right]\left[L_{\ell'} \cdot H\right] + h.c \tag{26}$$

In this formulation, the ℓ index refers to a flavour index and the WILSON coefficient has to be chosen so that the masses after breaking coincide with the experimental measurements. This operator is unique, it is the only $D = 5$ operator satisfying the gauge invariance of each gauge group: vanishing hypercharge, invariant contractions for both SU groups. To obtain the masses, we have to use the definitions of the HIGGS and lepton doublets as well as the $\mathrm{SU}(2)_L$ invariant product: $H \cdot \overline{L_\ell^c} = H^i \epsilon_{ij} \overline{L_\ell^{cj}}$

with $\epsilon_{12} = +1$. Going through electroweak breaking in the unitary gauge, the Higgs field develops a vacuum expectation value that we expand in the unitary gauge using the mean value v and the dynamical field h:

$$H \sim \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \tag{27}$$

The expansion of the Lagrangian yields:

$$\begin{aligned}\mathcal{L}_5 &= \frac{C^{(5)}_{\ell\ell'}}{\Lambda}\left[H^i\epsilon_{ij}\overline{L^{cj}_{\ell}}\right]\left[L^i_{\ell'}\epsilon_{ij}H^j\right] + h.c \\ &= -\frac{C^{(5)}_{\ell\ell'}v^2}{2\Lambda}\overline{\nu^c_\ell}\nu_{\ell'} - \frac{C^{(5)}_{\ell\ell'}v}{\Lambda}h\overline{\nu^c_\ell}\nu_{\ell'} - \frac{C^{(5)}_{\ell\ell'}}{2\Lambda}hh\overline{\nu^c_\ell}\nu_{\ell'} + h.c\end{aligned} \tag{28}$$

The second and third terms generate single and double Higgs couplings with neutrinos of flavours ℓ and ℓ'. This is a very strong prediction because in the Standard Model, neutrinos are not able to couple with the Higgs boson. The predicted total width of the Higgs boson being around 4.1 MeV, the partial width associated to these high energy interactions has to be small so that it fits into the uncertainties of the measured width that is already very narrow, predicting long-lived particles. The first term then generates a Majorana mass matrix:

$$m_{\ell\ell'} = \frac{C^{(5)}_{\ell\ell'}v^2}{\Lambda} \tag{29}$$

The purpose of this matrix is to be able to provide information about the masses of the physical neutrinos through its eigenvalues after rotation into the mass basis [14]. The value of the Wilson coefficient depends on the mass we want to generate. Let's consider the electron neutrino, whose mass is supposed to be lower than 0.07 eV. We consider the vacuum expectation value of the Higgs boson to be $v = 246$ GeV. The mass matrix element is then constrained:

$$m_{ee} = \frac{C^{(5)}_{ee}v^2}{\Lambda} < 0.07\ \text{eV} \Longrightarrow \frac{\Lambda}{C^{(5)}_{ee}} > 8.65\cdot 10^{14}\ \text{GeV} \tag{30}$$

If we want to take into account the interactions with Goldstone bosons generated by symmetry breaking:

$$H \sim \frac{1}{\sqrt{2}}\begin{pmatrix} -i\sqrt{2}G^+ \\ v+h+iG^0 \end{pmatrix} \tag{31}$$

Then, the full Lagrangian density for the Weinberg operator becomes:

$$\begin{aligned}&\mathcal{L}_5 - i\frac{C^{(5)}_{\ell\ell'}}{\Lambda}G^0(v+h)\overline{\nu^c_\ell}\nu_{\ell'} \\ &\quad - i\frac{C^{(5)}_{\ell\ell'}}{\sqrt{2}\Lambda}G^+(v+h)\left(\overline{\nu^c_\ell}\ell' + \overline{\ell^c}\nu_{\ell'}\right) \\ &\quad + \frac{C^{(5)}_{\ell\ell'}}{\sqrt{2}\Lambda}G^0G^+\left(\overline{\nu^c_\ell}\ell' + \overline{\ell^c}\nu_{\ell'}\right) \\ &\quad + \frac{C^{(5)}_{\ell\ell'}}{2\Lambda}\left(G^0G^0\overline{\nu^c_\ell}\nu_{\ell'} + 2G^+G^+\overline{\ell^c}\ell'\right) + h.c\end{aligned} \tag{32}$$

This Lagrangian leads to the Weinberg operator being applicable to meson and lepton decays, and establishes a road map to studying its behavior [15].

IV. PHENOMENOLOGY OF MAJORANA NEUTRINOS

This part is dedicated to trying to determine how to probe the nature of neutrinos and explore some of the underlying problems based on a model of Majorana neutrinos.

A. Mass mixing

Neutrinos change flavour while propagating, which is a direct consequence of the gauge eigenstates not being aligned with the mass eigenstates. Lepton mixing is described in the weak basis where the charged-lepton mass matrix is diagonal (with eigenstates e, μ and τ) so that the mixing matrix relates the neutrino weak-eigenstates ν_ℓ, $\ell = e, \mu, \tau$ and the neutrino mass eigenstates ν_i, $i = 1, 2, 3$. Each flavour (weak) eigenstate can be written as a combination of mass eigenstates, and the Pontecorvo-Maki-Nakagawa-Sakata matrix with components $U_{\ell i}$ contains the amplitudes of mass eigenstates in terms of flavour.

$$\begin{pmatrix}\nu_e\\ \nu_\mu \\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}, \qquad \nu_\ell = \sum_{i=1}^{3} U_{\ell i}\nu_i \tag{33}$$

The elements of the transition matrix are often parameterized by three mixing angles θ_{12}, θ_{23} and θ_{13} and three CP-odd phases δ, α and β [16]. These angles are the so-called Majorana phases, and if they are physically observable, they are currently unconstrained. We use c_{ij} and s_{ij} to denote $\cos(\theta_{ij})$ and $\sin(\theta_{ij})$:

$$U_{\ell i} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\alpha} & 0 & 0\\ 0 & e^{i\beta} & 0\\ 0 & 0 & 1\end{pmatrix} \tag{34}$$

This matrix is almost made of three rotation matrices, except for the δ angle introduced to take into account parity violation, and the α and β angles that represent the possibility for neutrinos to be MAJORANA particles. These phases do not influence neutrino oscillations directly but are crucial for processes that violate lepton number conservation allowed by the MAJORANA type neutrinos. We define the mass eigenstates by considering squared mass-differences between the three eigenstates. Given the three neutrinos, we define three squared mass matrices and two of them are independent. We impose that ν_1 and ν_2 define the smallest squared mass difference and impose that ${m_2}^2 > {m_1}^2$. The last state ν_3 is the left-over state that can be lighter or heavier that ν_1 and ν_2. Most of the necessary parameters are nowadays determined, however, several crucial pieces are still missing: the neutrino mass hierarchy (if the third component is lighter or heavier), the magnitude of the δ CP-phase, the absolute scale of the neutrino mass . . . In the normal hierarchy NH, the third mass eigenstate is the heaviest neutrino and we can represent the flavour composition depending on the CP-angle. In the inverse hierarchy IH, the lightest neutrino is the third one and the squared mass differences are different. Choosing between the two mass hierarchies is a difficult challenge due to the difficulties to measure properties of neutrinos. Depending on the hierarchy considered, we can write the expression of the masses of the mass eigenstates [17], by introducing a parameter m_{min} which is the mass of the lightest neutrino. The expressions are given under the corresponding hierarchy :

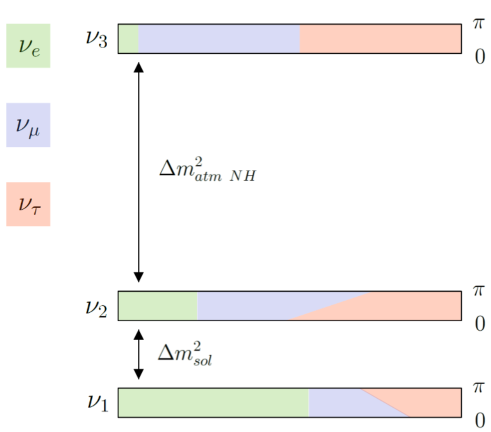

FIG. 1. Normal hierarchy

$$
\begin{aligned}
m_{1\ NH} &= m_{min} \\
m_{2\ NH} &= \sqrt{m_{min}^2 + \Delta m_{sol}^2} \\
m_{3\ NH} &= \sqrt{m_{min}^2 + \frac{\Delta m_{sol}^2}{2} + \Delta m_{atm\ NH}^2}
\end{aligned}
\tag{35}
$$

The flavours proportions varies with the eigenstate. In the inverse hierarchy :

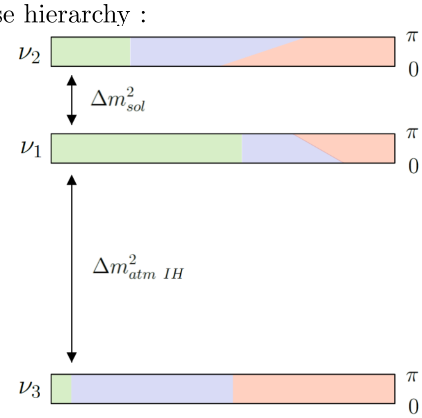

FIG. 2. Inverse hierarchy

$$
\begin{aligned}
m_{1\ IH} &= \sqrt{m_{min}^2 - \frac{\Delta m_{sol}^2}{2} + \Delta m_{atm\ IH}^2} \\
m_{2\ IH} &= \sqrt{m_{min}^2 + \frac{\Delta m_{sol}^2}{2} + \Delta m_{atm\ IH}^2} \\
m_{3\ IH} &= m_{min}
\end{aligned}
\tag{36}
$$

There could be several ways to determine the mass hierarchy, like looking at the sum of the three eigenmasses. If that sum is below the minimum mass of ~ 0.1 eV, the existence of the normal hierarchy is indicated. Similarly, in the study of the β decay, the mass of the electron neutrino can be theoretically determined using the mass eigenstates, and because they contribute with different weights to the quantum state, it could indicate a scenario. The numerical values are provided by the Particle Data Group. Cosmological observations tend to put several upper limits of about $1,0 - 2,0$ eV on the sum of neutrino masses [16], which is not enough to discriminate alone the mass hierarchy. What we have to investigate to go further are the reactions that could confirm the MAJORANA nature of neutrinos as well as associated observables that could indicate a hierarchy.

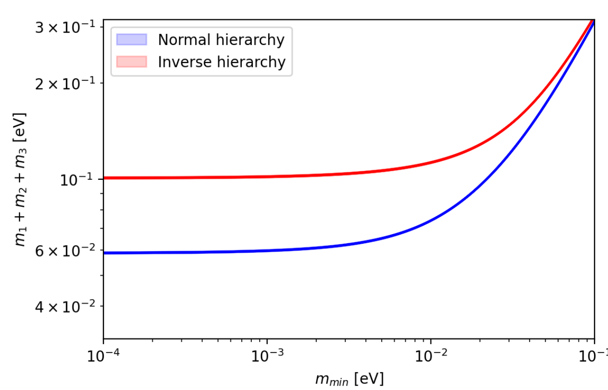

FIG. 3. $m_1 + m_2 + m_3$ as a function of the lightest neutrino mass for the two possible hierarchies

TABLE I. Parameters of the PMNS matrix used for the simulation

Δm^2_{sol}	$\Delta m^2_{atm\ NH}$	$\Delta m^2_{atm\ IH}$	δ
$(7.53 \pm 0.18) \cdot 10^{-5}$ eV^2	$(2.453 \pm 0.033) \cdot 10^{-3}$ eV^2	$(2.536 \pm 0.034) \cdot 10^{-3}$ eV^2	$1.36^{+0.20}_{-0.16}$ rad
s^2_{12}	s^2_{13}	$s^2_{23\ NH}$	$s^2_{23\ IH}$
$0.37^{+0.013}_{-0.012}$	$(2.20 \pm 0.07) \cdot 10^{-2}$	0.546 ± 0.021	0.539 ± 0.022

B. Neutrinoless double β decay

Studying the effective Standard Model tells us that neutral particles with a small MAJORANA mass is the first hint one would expect to observe from new high-scale physics because the relevant term in the Lagrangian generating MAJORANA neutrino masses is the only $D = 5$ operator (suppressed by only one power of some new high energy scale) consistent with the gauge symmetries. We need to figure out what type of process could be a sign that neutrinos are their own antiparticle. If it is the case, then we might imagine a process in which we produce an antineutrino alongside the associated charged lepton whose sign allows us to say an antineutrino has been produced, but this antineutrino would then interact as if it was a neutrino, producing another charged lepton [14], [18]. This process is only possible if the neutrino can change behavior and act as a particle, and this would produce a very distinct signature. Without neutrinos, the sum of the energy of the two electrons is a fixed quantity, and the energy spectrum for such a process simply becomes a peak at a defined energy. This would allow us to distinguish a regular double β decay from the neutrinoless version and then deduce the nature of the involved neutrino. This is for the case where the neutrinos are produced alongside electrons, but if we want to be more general, it is possible for each flavour of neutrinos.

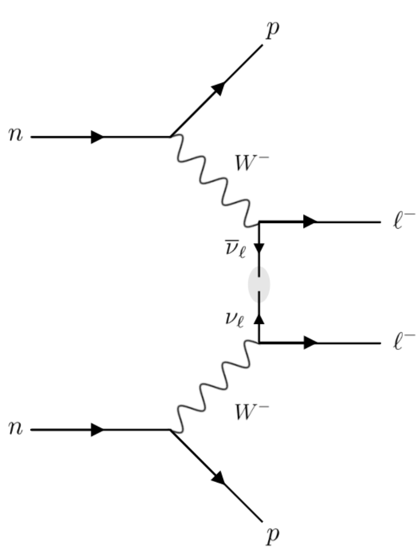

FIG. 4. FEYNMAN diagram for neutrinoless double β decay

What is different compared to a usual double β decay is the neutrino is not emitted but exchanged internally, implying it is a MAJORANA fermion. The observation of neutrinoless double β decay would indicate that lepton number conservation is violated, but we can also obtain information on the mass of this MAJORANA neutrino as the rate of neutrinoless double β decay is related to the effective MAJORANA mass of the neutrino. The WEINBERG operator involved in this process couples one massless left-handed neutrino with momentum p and flavour ℓ with the conjugate of a second neutrino of momentum p and flavour ℓ'. The neutrino current can be modeled as an unphysical MAJORANA neutrino N with mass m_N that contains the information about all the leptons that can be involved in the process:

$$m_N = \left| C^{(5)}_{ee} + C^{(5)}_{e\mu} + C^{(5)}_{e\tau} + C^{(5)}_{\mu\mu} + C^{(5)}_{\mu\tau} + C^{(5)}_{\tau\tau} \right| \frac{v^2}{\Lambda} \tag{37}$$

While it cannot directly determine the absolute neutrino mass scale, it is sensitive to the effective MAJORANA mass of the electron neutrino which we will define just after. By measuring or constraining this effective mass, experiments can provide insights into the neutrino mass hierarchy and potentially the absolute mass scale, when combined with other neutrino oscillation data.

C. Decay phenomenology

We assume for now that the two leptons involved are only of electron type, meaning that the process is made through a standard light neutrino exchange. There is a parameter of interest, which is the effective MAJORANA mass, encapsulating the contributions of different neutrino mass eigenstates to the decay process:

$$|m_{\beta\beta}| = \left| c^2_{12} c^2_{13} e^{2i\alpha} m_1 + s^2_{12} c^2_{13} e^{2i\beta} m_2 + s^2_{13} e^{-2i\delta} m_3 \right| \tag{38}$$

This parameter is quite powerful because it contains information about the mass eigenstates, the mass hierarchy, but also the MAJORANA phases α and β. The first thing we can do is investigating the effective MAJORANA mass as a function of the lightest neutrino mass for the two hierarchies. Then, we could also wonder what is the behavior of the effective mass depending on the sum of the three masses. This is interesting because cosmological observation puts limits on this mass and so can exclude some mass domain for us to investigate. In the case of normal hierarchy, the effective mass is distributed within a flat area between 10^{-3} and $5 \cdot 10^{-3}$ eV for a minimum mass under 10^{-3} eV while it seems it vanishes between 10^{-3} and 10^{-2} eV due to the combination of the Majorana phases. In the inverse hierarchy, the values are higher and the flat area spreads more. A vanishing effective mass does not imply that the theory suffers from dangerous fine tuning because the effective mass could assume a naturally small value that remains small after renormalization due to the chiral symmetry of fermions. A similar comment can be made on the second plot, except that they are dependent on the cosmological limit on the neutrino masses. Measuring an effective mass could then be used to discriminate the two hierarchies based on the accessible parameter space [17].

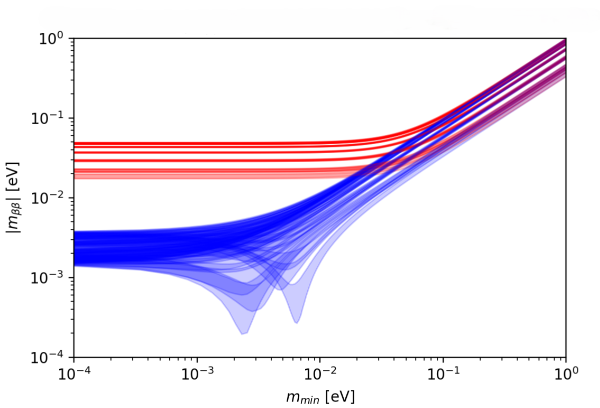

FIG. 5. Effective mass as a function of the minimal neutrino mass

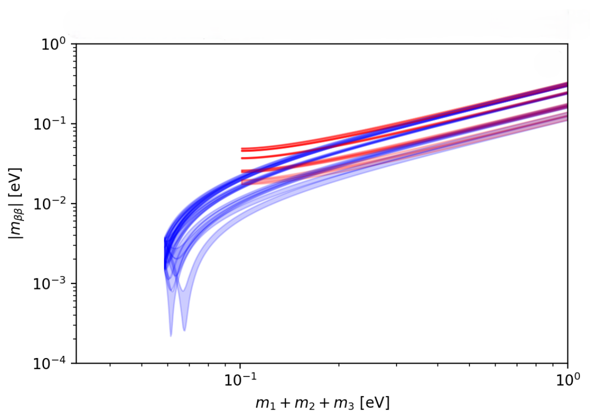

FIG. 6. Effective mass as a function of the summed eigenmasses

Then, what if there are additional sterile neutrinos ? We could assume a new contribution in the effective mass that is encoded the following way:

$$|m_{\beta\beta}| \longrightarrow |m_{\beta\beta} + \mathcal{M}_4| \tag{39}$$

$\mathcal{M}_4$ is a free parameter. As a result of adding new contributions to the effective mass, phenomenology gets completely turned out because the inverse hierarchy effective mass can now vanish compared to the normal hierarchy mass for some values of $\mathcal{M}_4$. The fourth contribution in the effective mass is not the unphysical neutrino introduced before. The see-saw mechanism involves new types of neutrinos called sterile, that would be more massive than the neutrinos we know and would not interact via weak interactions, and in the presence of sterile neutrinos, the neutrino mixing matrix extends beyond the standard matrix to include the mixing of sterile states, now including contributions from the mixing of the electron neutrino with both active and sterile states. These new mass states can contribute to the effective Majorana mass if they have a significant mixing with the electron neutrino and if they are Majorana particles. Sterile neutrinos at a high mass scale lead to the Weinberg operator after integrating out the heavy states. The effective scale Λ in the Weinberg operator is then related to the mass scale of these sterile neutrinos. If sterile neutrinos are light, they can still contribute to the Weinberg operator through their mixing with active neutrinos. In this case, the effective operator might receive corrections due to the presence of additional light states, modifying the neutrino mass matrix.

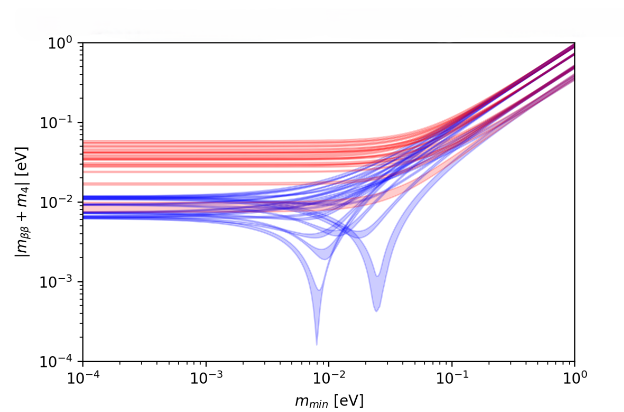

FIG. 7. Modified effective mass for $\mathcal{M}_4 = 10^{-2}$ eV

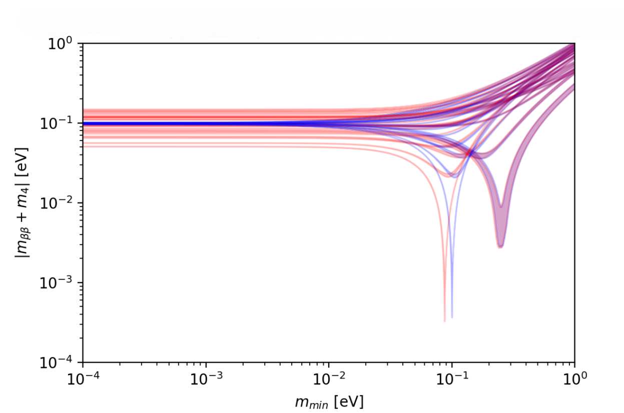

FIG. 8. Modified effective mass for $\mathcal{M}_4 = 10^{-1}$ eV

CONCLUSION

The aim of this study was to identify properly what problems are nowadays encountered when studying neutrinos, but also to draw the guideline to a possible study at colliders. The discovery of neutrinoless double β decay would be a solid proof that neutrinos are in fact MAJORANA particles, but also bring information about the hierarchy and the decay rate as it is related to the effective MAJORANA mass $m_{\beta\beta}$. In this article, we do not give information about the cross section of such an interaction, but we already know it should be very small. What can also be investigated is the possibility for a double decay with muon or tau neutrinos, the effective mass is directly deduced from the mass mixing matrix, and phenomenology is modified, leading to a completely different accessible parameter space. This result has to be manipulated carefully, because changing the type of neutrino changes the cross section but also the type of experiment to perform. It could also be possible that no nucleus is able to decay using these neutrinos because they deal with higher energies that can be out of the excitation spectrum. The study of the WEINBERG operator then predicts new interactions between neutrinos and the HIGGS bosons, which can be investigated (currently, the guess of this study would be 10^{-24} GeV) to obtain the partial width and try to detect any deviation from the Standard Model.

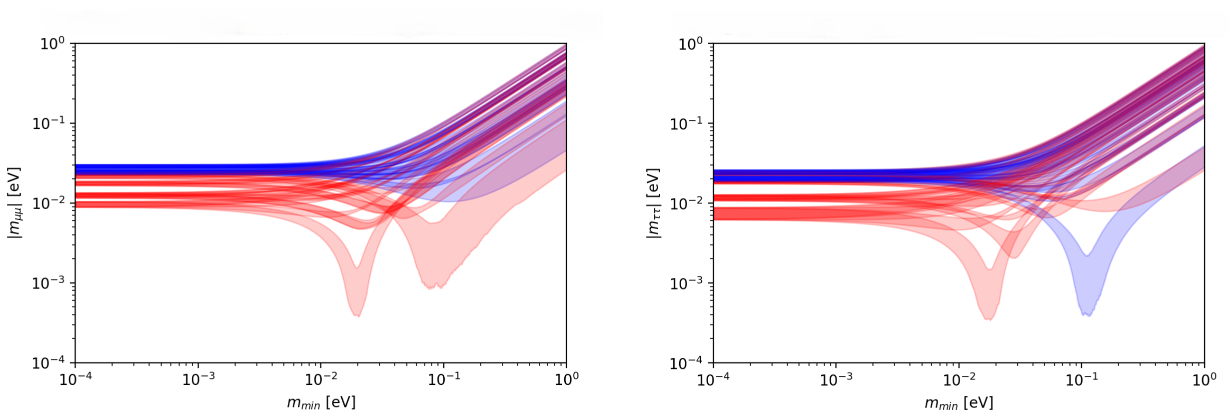

FIG. 9. Effective mass as a function of the minimal neutrino mass in the muon (a) and tau (b) case

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